IMPROVED LERAY-TRUDINGER INEQUALITY

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In this talk, we will present some recent results on Leray-Trudinger type inequalities that are closely related to Trudinger-Moser and Hardy inequalities. Let $n \geq 2$, $\Omega \subset \mathbf{R}^n$, be a bounded domain. The classical result of N. Trudinger asserts that, for any $q > n$, we have

$$\sup_{u \in \mathcal{D}} \frac{\|u\|_{L^q(\Omega)}}{q^{1-1/n}} < \infty \quad \text{where} \quad \mathcal{D} := \left\{ u \in W_0^{1,n}(\Omega) \ : \ \|\nabla u\|_{L^n(\Omega)} \leq 1 \right\},$$

The exponent $1 - 1/n$ is sharp, i.e., cannot be increased. From this result, one entails the exponential integrability of functions belonging to $\mathcal{D}$. If Ω contains the origin, setting $R_\Omega := \sup_{x \in \Omega} |x|$, for all $u \in W_0^{1,n}(\Omega)$ the critical case of Hardy's inequality holds true, i.e.

$$I_n[u;\Omega] := \int_\Omega |\nabla u(x)|^n \, \mathrm{d}x - \left(\frac{n-1}{n}\right)^n \int_\Omega \frac{|u(x)|^n}{|x|^n} X^n\left(\frac{|x|}{R_\Omega}\right) \mathrm{d}x \geq 0,$$

where $X(t) := (1 - \log t)^{-1}$, $t \in [0,1]$. One can now wonder whether exponential-type integrability holds also for functions satisfying $I_n(u,\Omega) \leq 1$. The Leray-Trudinger inequality ensures this type of integrability up to a logarithmic correction. The plan of the talk is to present the origin and the history of the problem and to present an optimal analogous of the Trudinger's inequality for functions in $\mathcal{H} := \left\{ u \in W_0^{1,n}(\Omega) \;\middle|\; I_n[u;\Omega] \leq 1 \right\}$. The presentation is based on joint works with G. di Blasio and G. Psaradakis.

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